0

NASA TECHNICAL MEMORANDUM NASA TM-76237

EVALUATION OF ORBITS WITH INCOMPLETE KNOWLEDGE OF THE MATHEMATICAL EXPECTANCY AND THE MATRIX OF COVARIATION OF ERRORS

B. Ts. Bakhshiyan, R. R. Nazirov, and P. E. El'yasberg

Translation of "Otsenki orbit pri nepolnom znanii matematicheskogo ozhidaniya i matritsy kovariatsii oshibok", Academy of Sciences USSR, Institute of Space Research, Moscow, Report Pr-360, 1977, pp. 1-18

(NASA-TM-76237) EVALUATION OF ORBITS WITH INCOMPLETE KNOWLEDGE OF THE MATHEMATICAL EXPECTANCY AND THE MATRIX OF COVARIATION OF ERRORS (National Aeronautics and Space Administration) 19 p HC A02/MF A01 CSCL 03B G3/90

N81-21984

Unclas 41933

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON, D.C. 20546 AUGUST 1980

STANDARD TITLE PAGE

1. Report No. NASA TM-76237	2. Government Accession No.	3. Recipient's Catalog No.
4. Title and Subtitle EVALUATION OF ORBITS WITH INCOMPLETE KNOWLEDGE OF THE MATHEMATICAL EXPECTANCY AND THE MATRIX OF COVARIATION OF ERRORS		5. Report Date August 1980
		6. Performing Organization Code
7. Author(s) B. Ts. Bakhshiyan, R. R. Nazirov, and P. E. El'yasberg		8. Performing Organization Report No.
		10. Work Unit No.
9. Performing Organization Name and Address Leo Kanner Associates Redwood City, California 94063		11. Contract or Grant No. NASw-3199
		13. Type of Report and Period Covered Translation
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration, Washington, D. C. 20546		14. Sponsoring Agency Code

15. Supplementary Notes

Translation of "Otsenki orbit pri nepolnom znanii matematicheskogo ozhidaniya i matritsy kovariatsii oshibok", Academy of Sciences USSR, Institute of Space Research, Moscow, Report Pr-360, 1977, pp. 1-18

16. Abstract

Examined herein is the problem of selection of the optimal algorithm of filtration and the optimal composition of the measurements, assuming that the precise values of the mathematical expectancy and the matrix of covariation of errors are unknown.

The optimal algorithm of filtration may be utilized for making some parameters more precise (for example, the parameters of the gravitational fields) after preliminary determination of the elements of the orbit by a simpler method of processing (for example, the method of least squares).

17. Key Words (Selected by Author(s))	18. Distribution Statement Unclassified-Unlimited

19. Security Classif. (of this report) Unclassified	20. Security Classif. (of this page) Unclassified	21. No. of Pages	22.

EVALUATION OF ORBITS WITH INCOMPLETE KNOWLEDGE OF THE MATHEMATICAL EXPECTANCY AND THE MATRIX OF COVARIATION OF ERRORS

B. Ts. Bakhshiyan, R. R. Nazirov, and P. E. El'yasberg

Examined herein is the problem of selection of the optimal algorithm of filtration and the optimal composition of measurements, assuming that the precise values of the mathematical expectancy and the matrix of covariation of errors are unknown. In this case, optimization is carried out from the condition of attainment of a maximum guaranteed reliability H_{gar} of determination of the scalar parameter. H_{gar} is understood to mean the minimum value of reliability, with a given maximum error, in a set of possible laws of distribution of the summary errors of the utilized mathematical model of motion and measurements. This set is determined by means of the application of some limitations to the mathematical expectancy and the value of the elements of the covariation matrix of errors. The expression for H_{gar} may be utilized for obtaining the guaranteed evaluation of the accuracy with determination of the orbits by a random linearizable algorithm of filtration. It is shown that the problem of optimization of H_{gar} amounts to the solution of some problem of quadratic programming. The optimal algorithm of filtration, obtained in this case, may be utilized for making some parameters more precise (for example, the parameters of the gravitational fields) after preliminary determination of the elements of the orbit by a simpler method of processing (for example, the method of least squares). /2*

1. Evaluation of Accuracy of Determination of Orbits /3

The determination of the orbits amounts to the obtaining of an evaluation of $\hat{q}$ of some vector $q=\{\overline{q_1, q_m}\}$, which makes it possible to calculate the motion in accordance with the given mathematical model [1]. This evaluation is described as the function

*Numbers in the margin indicate pagination in the foreign text.

$$\hat{q} = \hat{q}(\tilde{d}) \qquad (1)$$

of the vector of the measurements $\tilde{d}=\{\overline{d_1,d_n}\}$, which may be represented in the form

$$\tilde{d} = d(q) + \xi , \qquad (2)$$

where ξ is the vector of errors of the initial data, equal to the sum of the errors of the measurements and the model (2) [1]. The method of least squares (m.n.k.) is the most widespread algorithm of evaluation, with which

$$\hat{q} = \arg\min_{q} \left[\tilde{d} - d(q)\right]^T K^{-1} \left[\tilde{d} - d(q)\right] . \qquad (3)$$

Here, K is the non-negatively determined matrix. The following assumptions on the mathematical expectancy of error ξ and its covariation matrix are usually utilized during the derivation of the statistical properties of the evaluation of (3):

$$E(\xi) = 0 , \quad D(\xi) = \sigma^2 K . \qquad (4)$$

With the assumptions in (4), the evaluation of (3), in many cases, is justifiable [1], i.e., with an increase in the number of utilized measurements, it coincides, in probability, with the true value of q_{ist} of the parameters $q: \hat{q} \xrightarrow{p} q_{ist}$ with $n \to \infty$. In reality, the assumptions in (4) are never precisely fulfilled, and, with sufficiently large n, the evaluation of the accuracy of determination of the parameter q, obtained on their basis, proves to be unjustifiably optimistic. Depicted in figure 1 is the dependence of the root-mean-square value of $\sigma(\hat{\ell})$ of the evaluation $\hat{\ell}$ of some scalar parameter ℓ on the number n of utilized measurements. In this case, the theoretical dependence, which corresponds to the assumptions in (4), is depicted by the dotted line, and the depen- /4

dence obtained in practice is depicted by the solid line. Such a situation is inadmissible during the solution of problems of navigation or determination of important physical constants, as well as during the selection of the optimal strategy of determination of an orbit and its correction.

2. Guaranteed Approach in the Problem of Evaluation of Accuracy

In connection with what has been set forth, the problem arises of obtaining reliable (although less precise) evaluations of the accuracy of determination of the orbit parameters. For this purpose, the problem of obtaining and optimizing the guaranteed evaluations of accuracy, with incomplete information on the statistical characteristics of the errors in the inital data, is examined in a number of studies [1-6]. The present study is a generalization and further development of the methods of obtaining and utilizing guaranteed evaluations of the accuracy. In this case, the results, obtained earlier, may be viewed as its partial cases.

Let the set $\mathcal{F}$ of possible functions of determination $F(\xi)$ of the error ξ be given

$$F(\xi) \in \mathcal{F}, \tag{5}$$

and H (or α) is some scalar characteristic, to an increase (or decrease) in which corresponds an increase in the accuracy of determination of the orbit. We will determine the guaranteed value of these characteristics from the inequalities /5

$$H_{gar} \leq \inf_{F(\xi)\in\mathcal{F}} H \quad \left(\alpha_{gar} \geq \sup_{F(\xi)\in\mathcal{F}} \alpha\right). \tag{6}$$

If equalities are achieved in (6), then we will state that H_{gar} and α_{gar} are strict guaranteed characteristics. The characteristics in (6) are determined for the given algorithm of filtration. It is natural to pose the problem of finding of the algorithm

which is optimal in the sense of an increase (or decrease) in the guaranteed (desirably strict) characteristics, i.e., to seek it in a set S of given algorithms of filtration

$$\sup_{S} H_{gar} \quad \text{or} \quad \inf_{S} \alpha_{gar} \,. \tag{7}$$

In order to calculate the extremums in (6) and (7), it is necessary to prescribe the sets $\mathcal{F}$ and S and the characteristics of accuracy.

3. The Set $\mathcal{F}$

We will assume that the conditions which determine the set $\mathcal{F}$ contain the following limitations on the mathematical expectancies $E(\xi_i)$ and the elements D_{ij} of the covariation matrix of $D(\xi)$ $(i,j=\overline{1,n})$

$$|E(\xi_i)| \leq M_i \,, \tag{8}$$

$$D^*_{ij} - v_{ij} \leq D_{ij} \leq D^*_{ij} + v_{ij} \,, \quad i \neq j \,; \quad 0 \leq D_{ii} \leq D^*_{ii} + v_{ii} \,, \tag{9}$$

where M_i, D^*_{ij}, $v_{ij} \geq 0$ are the elements of the given matrices M, D, V of dimensions 1 x n, n x n, n x n. In addition, we will indicate the results which take place for the case when, instead of (9), similar limitations are given on the coefficients of correlation $k_{ij}=D_{ij}/\sqrt{D_{ii}D_{jj}}$ and dispersion: /6

$$k^*_{ij} - w_{ij} \leq k_{ij} \leq k^*_{ij} + w_{ij} \,, \quad i \neq j \,; \quad \delta_i^2 \leq D_{ii} \leq \Delta_i^2 \,, \tag{10}$$

where k^*_{ij}, w_{ij} are the elements of the given matrices K*, W, in which $k^*_{ii}=1$, $w_{ii}=0$, and $i=\overline{1,n}$.

In addition to conditions (8)-(10), which we will call the basic conditions, the set $\mathcal{F}$ may be determined by additional limi-

tations as well, which can be placed on the function of distribution of the errors (for example, the form of this function may be prescribed).

We will give examples of the representation of the set $\mathcal{F}$ using (8)-(10). With M=0, V=0, we obtain the classic assumptions in (4), on the basis of which we construct the evaluations of the method of least squares [7]. If $M \neq 0$, $D^* = V = 0$, then the set $\mathcal{F}$ is characterized by the conditions $|\xi_i| \leqslant M_i$ (the "skirting" scheme) [2]. The cases $k_1 \leqslant k_{ij} \leqslant k_2$, $D_{ii} = \Delta_i^2$, and $|k_{ij}| = k \leqslant 1$, $D_{ii} \leqslant \Delta_i^2$ are examined in [1,5,6].

4. Characteristics of Accuracy and Their Guaranteed Values

In the capacity of the characteristic of accuracy H, we will take the probability of the fact that the error

$$\delta\ell = \ell(\hat{q}) - \ell(q) = \ell\left[\hat{q}(\tilde{d})\right] - \ell(q) \tag{11}$$

of determination of some scalar parameter $\ell(q)$ does not exceed the given magnitude of $\alpha > 0$, according to the modulus, i.e.,

$$H = P(|\delta\ell| \leqslant \alpha). \tag{12}$$

If H is given with reliability, then one can view $\alpha = \alpha(H)$ as the characteristic of accuracy. The solution of problems (6), (7) for the introduced characteristics H and α, with the conditions /7
given below, are equivalent [1]. Therefore, we will subsequently make use of the characteristic H.

We will calculate H_{gar} for the algorithms of filtration (1), which satisfy the following conditions.

1. If $\xi = 0$, then $\delta\ell = 0$ for all q from the region of its possible values. According to (1), (2), and (11), this condition may be

written in the form

$$\ell \{ \hat{q} [d(q)] \} = \ell(q) . \tag{13}$$

If (13) is fulfilled, then we will state that the algorithm of filtration (1) possesses the property of unbiasedness.

2. The function $\ell[\hat{q}(\tilde{d})]$ is linearizable at some prior known point q_0, i.e.,

$$\ell[\hat{q}(\tilde{d})] = \ell(q_0) + X[\tilde{d} - d(q_0)] , \tag{14}$$

where $X=(\overline{x_1, x_n})$ is a matrix of dimensions 1 x n. Hence, taking (13) into account, it is not difficult to obtain

$$\delta \ell = X \xi . \tag{15}$$

We will find H_{gar} for two cases.

A. In addition to conditions (8), (9), there is no additional information on $F(\xi)$, which makes it possible to find the law of distribution of the error $\delta\ell$.

B. The distribution $\delta\ell$ may be considered normal. (Such an assumption corresponds to the truth if, for example, the errors ξ satisfy the conditions of the central limit theorem).

For case A, utilizing Chebyshev's inequality, we obtain the following from (6) for the strict guaranteed characteristic:

$$H_{gar} = \max_{F(\xi)\in\mathcal{F}} H = 1 - \left[E(\delta\ell^2)_{gar} \right] / \alpha^2 , \tag{16}$$

where

$$E(\delta\ell^2)_{gar} = m_{gar}^2 + D_{gar}; \quad m_{gar} = \max_{F(\xi)\in\mathcal{F}} [E(\delta\ell)], \; D_{gar} = \max_{F(\xi)\in\mathcal{F}} \Delta(\hat{\ell}) \tag{17}$$

/8

are strict guaranteed characteristics.

In case B, the reliability of (12) is equal to [1]

$$H = H(\alpha, m, D) = \Phi\left(\frac{\alpha - m}{\sqrt{D}}\right) - \Phi\left(\frac{\alpha + m}{\sqrt{D}}\right) , \tag{18}$$

where $m = |E(\delta\ell)|$, $D = D(\hat{\ell})$, $\Phi(X) = \frac{1}{\sqrt{2\pi}} \int_0^X \exp\left(-\frac{t^2}{2}\right)dt$ ..

We will assume that $\alpha > m$, since, in the opposite case, $H(\alpha, m, D) \leqslant 0.5$, which is unacceptable in practical problems of the determination of orbits. Then, the derivatives of function (18), according to m and D, are negative, in virtue of which we obtain

$$\min_{F(\xi)\in\mathcal{F}} H(\alpha, m, D) = H_{gar} = H(\alpha, m_{gar}, D_{gar}) , \tag{19}$$

where m_{gar}, D_{gar} are found from (17).

Thus, in both cases A and B, for the calculation of H_{gar}, it is necessary to find the strict guaranteed characteristics (17). According to (8),

$$m_{gar} = X_{+}M , \tag{20}$$

where $X_{+} = (|x_1|, |x_n|)$. During the calculation of D_{gar}, it should be taken into account that the acceptable matrices $D(\xi)$ should satisfy not only conditions (9), but should also be non-negatively determined. In this connection

$$D_{gar} \leqslant X^T D^* X + X_{+} V X_{+}^T , \tag{21}$$

where the right-hand portion in (21) is the maximum of $D(\hat{\ell})$, with the conditions in (9).

For D and V, the necessary and sufficient conditions may be obtained so that in (21), with any X, the equality would be fulfilled. It follows from them, specifically, that the conditions of non-negative definiteness of the matrices D^*+Z and $V-Z$ are sufficient with some value of the diagonal matrix Z, while the non-negative definiteness of the matrix D^*+V is necessary. /9

Now, let the limitations on the covariations be given in the form of (10), and, under these conditions, either $w_{ij} \geq |k^*_{ij}|$, $i \neq j$, or $\delta_i = \Delta_i$. In this case, we will adopt the designations $\Lambda = \mathrm{diag}[\Lambda_1, \Lambda_n], \Lambda D^* = \Lambda K^*$. Then, one can show that the maximum of $D(\hat{\ell})$, with the conditions in (10), may also be written in the form of the right-hand portion of (21). This makes it possible to subsequently view only that case when the conditions in (9) are given, since similar results will occur with limitations in the form of (10).

Sufficient conditions for the implementation of the equality in (21) are fulfilled for all of the examples enumerated in paragraph 3. Subsequently, we will also assume that these conditions are fulfilled, and there is an equal sign in (21).

5. Optimization of the Guaranteed Reliability

Relationships (16)-(21) give strict guaranteed values of the characteristic H for cases A and B from paragraph 4. Therefore, in both variants, problem (7) is reduced to maxi-mini optimization

$$\max_{X} H_{\text{gar}} = \max_{X} \min_{F(\xi) \in \mathcal{F}} H. \tag{22}$$

Here, the maximum is taken according to the line X, which, according to (14), determines the algorithm of filtration and satisfies the condition of unbiasedness (13) of this algorithm.

We will assume that the functions $d(q)$ and $\ell(q)$, similar to (14), are linear /10

$$d(q) = d(q_0) + A(q - q_0) ,$$
$$\rho(q) = \rho(q_0) + C(q - q_0) . \tag{23}$$

where A and C are known matrices of dimensions n x m and 1 x n. From (11), (14), and (23), we obtain that the condition of unbiasedness (13) takes on the form

$$XA = C . \tag{24}$$

For case A, problem (22), according to (10) and the obtained strict characteristics, amounts to finding

$$\min_{XA = C} E(\delta\rho^2)_{gar} \quad \min\{XD^{*}X^{T} + X_{+}(MM^{T} + I)X_{+}^{T}\} . \tag{25}$$

Similar to that, as was done in [5], minimization in (25), by means of substitution of the variables

$$x_i = x_i' - x_i'' , \quad |x_i| = x_i' + x_i'' , \quad x_i', x_i'' \geq 0, \quad i = \overline{1, n} \tag{26}$$

is brought to the problem of quadratic programming, the solution of which satisfies the conditions $x_i'x_i''=0$, which are necessary for the correctness of the substitution (26).

We will now switch to case B. In order to find the maximum in (22), we will examine first the auxiliary problem

$$\max H_{gar} \tag{27}$$

with the conditions in (24) and the additional limitation

$$m_{gar} = \mu , \tag{28}$$

where m_{gar} is determined from (20), and μ=const.

If the solution of this problem will be known for all of the

acceptable μ

$$\mu \in |m_0, \dot{m}_k|, \quad m_0 = \min_{XA=C} m_{gar}, \quad m_k < \alpha, \tag{29}$$

then the maximum (22) is found by means of unidimensional optimization according to μ in the interval in (29). Since the derivative of H_{gar} according to D_{gar} is negative, the maximization in (27) amounts to the finding of

$$f(\mu) = \min D_{gar} = \min \{X D^* X^T + X_+ V X_+^T\} \tag{30}$$

with the conditions (24) and (28). Thus, problem (22), in case B, is reduced to unidimensional maximization, according to μ, of the function $H(\alpha,\mu,f(\mu))$ in the interval (29) (see fig. 2). The finding of the function $f(\mu)$ in this segment, after the substitution (26), amounts to the problem of quadratic programming with the parameter μ in the right-hand portion of condition (28). The last problem is of the very same order of complexity as the problem of quadratic programming [8,9]. During its solution, the interval (29) is subdivided into a finite number of segments, inside of each of which the solution is achieved on one and the same basis, and $f(\mu)$ is an arc of a parabola. One can show that, in each of these segments, there is no more than one critical point of the function $H(\alpha,\mu,f(\mu))$, and this point is found by means of a search of the root of some increasing function $f(\mu)$ (see fig. 2). A simple graphic method is developed along with the described analytical method for finding the optimal μ.

Thus, for the examined practically important cases A and B, the solution of the problem of optimization of the guaranteed reliability may be obtained using the well-developed algorithms of quadratic programming.

6. Determination of the Radial Velocity According to Range Measurements

In the capacity of an example, we will examine the case when

we must determine the radial velocity v of a cosmic object at a moment in time $t=0$, according to measurements of the ranges d_i from a measuring point at the moment t_i. In this case, it is assumed that some prior dependence $a(t)$ of the radial velocity on time is known. Then, relationship (2) has the form

$$\tilde{d}_i = r + vt_i + \int_0^{t} dt \int_0^{t} a(\tau)\, d\tau + \xi_i , \qquad (31)$$

where r is the radius with $t=0$, and ξ_i is the sum of the error of the measurements and the error evoked by the inaccuracy of knowledge of the radial acceleration, which we will assume is a non-random magnitude, not exceeding the given number w. We will also assume that the mathematical expectancy of errors of the measurements and the coefficients of correlation between them, for all errors, are limited according to the modulus by the given magnitudes $M>0$ and $K>0$, and the dispersions do not exceed σ^2. Then, for the summary error ξ, we obtain

$$|E(\xi_i)| \leq M + wt_i^2/2 , \quad D(\xi_i) \leq \sigma^2, \quad |k_{ij}| \leq k . \qquad (32)$$

We will assume that, at each moment t_i, repetition of measurements is allowed, so that their total number would not exceed some n.

One can show that the optimal solution of problem (22), for cases A and B, consists of the conduct, according to n/2, of measurements at some moments $t_1=T$ and $t_2=-T$. In this case, max H_{gar} may be represented as a function of the dimensionless parameters $\bar{a}=\alpha/\beta$, $\bar{M}=M/\gamma$, where $\beta^2=\min_x nE(\delta\ell^2)_{gar}$ and $\gamma^2=[(1-k)/n+k]/\sigma^2$. /13
Given in figure 3 are the graphs of the dependences max $H_{gar}(\alpha)$, found for values of $\bar{M}=0,1,2$; in this case, the solid lines correspond to case A, and the dotted lines to case B. It is evident from figure 3 that the switch from case A to case B involves a slight decrease in the maximum error α (no greater than 17% with equal $H_{gar} \leqslant 0.999$).

Calculations were carried out for the examined example, which showed that, with filtration of the method of least squares, with equal weights in some segment $[-T_{MNK}, T_{MNK}]$, the obtained H_{gar} only then differs little from its optimal value, when the value of T_{MNK} is correctly selected. With other T_{MNK}, the guaranteed reliability decreases sharply with evaluation of the method of least squares.

7. Practical Utilization of the Optimal Algorithms of Filtration

The method, proposed in the present study, for constructing the optimal algorithm of filtration makes it possible to increase the guaranteed accuracy of determination of the orbit, as compared with the results obtained using the classical algorithms, based on the given values of the mathematical expectancy and the co-variation matrix of errors. However, it possesses a number of shortcomings. The basic of these are:

-considerably greater (as compared with classical algorithms) labor consumption of calculations;

-dependence of algorithm on selection of parameter being evaluated;

-optimalness of algorithm only with linear posing of the problem, leading to the necessity of solution of nonlinear problems by the method of iterations. /14

The indicated circumstances evidently limit the utilization of the proposed optimal algorithm of filtration for the solution of operational problems of determination of the parameters of motion of a space vehicle. However, with secondary processing of the data of trajectory measurements, in order to determine the parameters which have independent scientific value (elements of orbits of natural heavenly bodies, parameters of gravitational fields of these bodies, and so on), the use of the indicated optimal algorithm of filtration may prove advisable. In this case, the following order of processing of the measurements may be proposed.

1. The preliminary orbit, which is utilized as the reference orbit for construction of a linearized mathematical model (2), is

determined by one of the classical methods.

2. According to the results of the study of the utilized measurement system and mathematical model, and also on the basis of the correlation analysis of the discrepancies of the system of conditional equations, obtained in the process of preliminary determination of the orbit, one can find the possible limits of change in the mathematical expectancy and the covariation matrix of errors. According to these data, conditions are selected which determine the set $\mathcal{F}$, and the exclusion of anomalous measurements is carried out.

3. The value of each of the interesting parameters ℓ is made more precise, using the corresponding optimal algorithm of filtration, obtained from the solution of the problem of quadratic programming.

4. The guaranteed characteristics of accuracy of the obtained evaluations of $\hat{\ell}$ are determined according to formulas (20) and (21).

REFERENCES

1. El'yasberg, P. Ye., Opredelenie dvizheniya po rezul'tatam izmereniy [Determination of Motion According to the Results of Measurements], Moscow, "Nauka", 1976.

2. Lidov, M. L., "K apriornym otsenkam tochnosti opredeleniya parametrov po metodu naimen'shikh kvadratov" [Prior Evaluations of the Accuracy of Determination of Parameters According to the Method of Least Squares], Kosmicheskie issledovaniya 2, 5, 713 (1964).

3. Belousov, L. Yu., "Opredelenie optimal'nykh momentov izmereniya" [Determination of Optimal Moments of Measurement], Kosmicheskie issledovaniya 7, 1, 28 (1969).

4. Bakhshiyan, B. Ts., El'yasberg, P. Ye., "Vybor optimal'noy strategii opredeleniya orbit" [Selection of Optimal Strategy of Determination of Orbits], Avtomatika i telemekhanika, 3, 16 (1970).

5. Bakhshiyan, B. Ts., "Nekotorye zadachi otsenki tochnosti prognozirovaniya parametrov traektorii i algoritmy ikh resheniya" [Some Problems of Evaluation of the Accuracy of Forecasting of Parameters of the Trajectory and Algorithms for their Solution], Kosmicheskie issledovaniya 12, 6, 811 (1970).

6. Bazhinov, I. K., Aleshin, V. I., Pochukaev, V. N., Polyakov, V. S., Kosmicheskaya navigatsiya [Space Navigation], Moscow, "Mashinostroenie", 1975.

7. Rao, S. R., Lineynye statisticheskie metody i ikh primenenie [Linear Statistical Methods and their Application], Moscow, "Nauka", 1968.

8. Boot, J., Quadratic Programming, Amsterdam, 1964.

9. Van de Pann, Methods for Linear and Quadratic Programming, New York-Amsterdam, 1975.

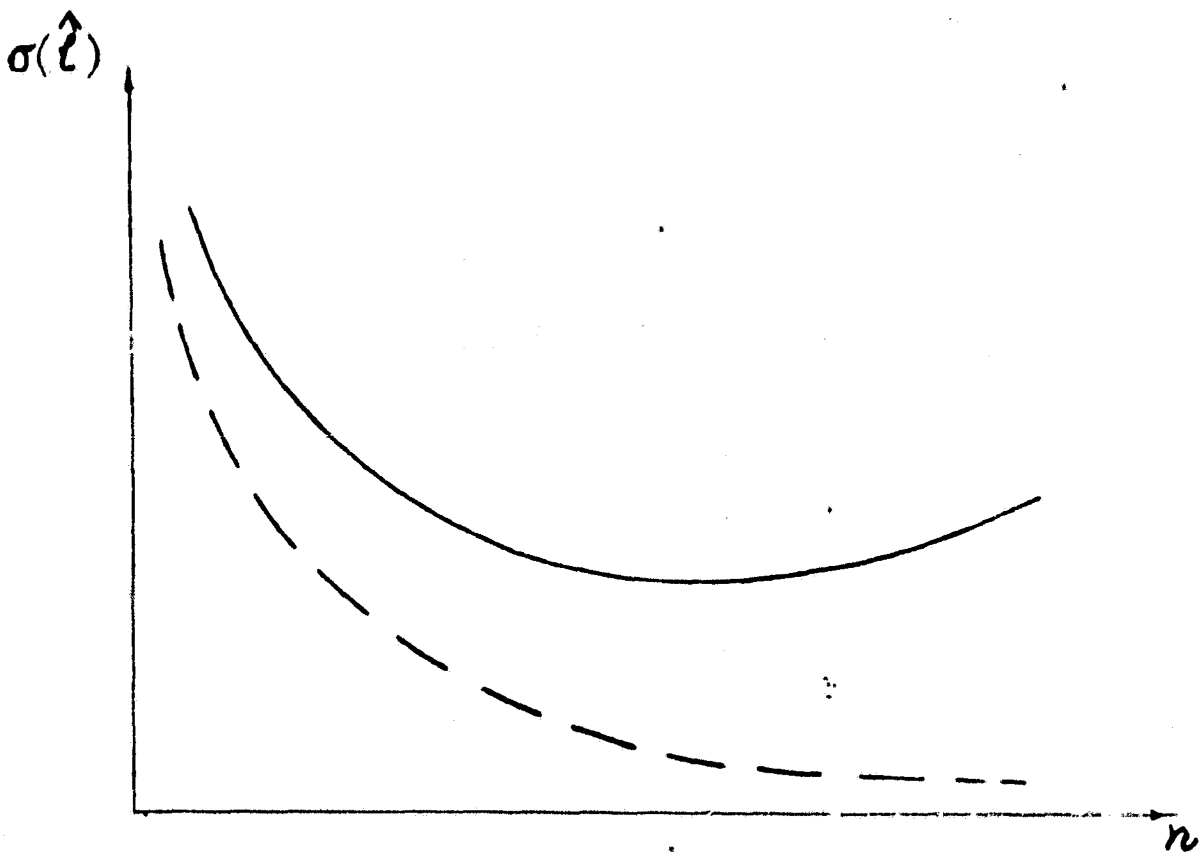

FIG. 1

Theoretical and practical dependences of $\sigma(\hat{\ell})$ on the number n of utilized measurements

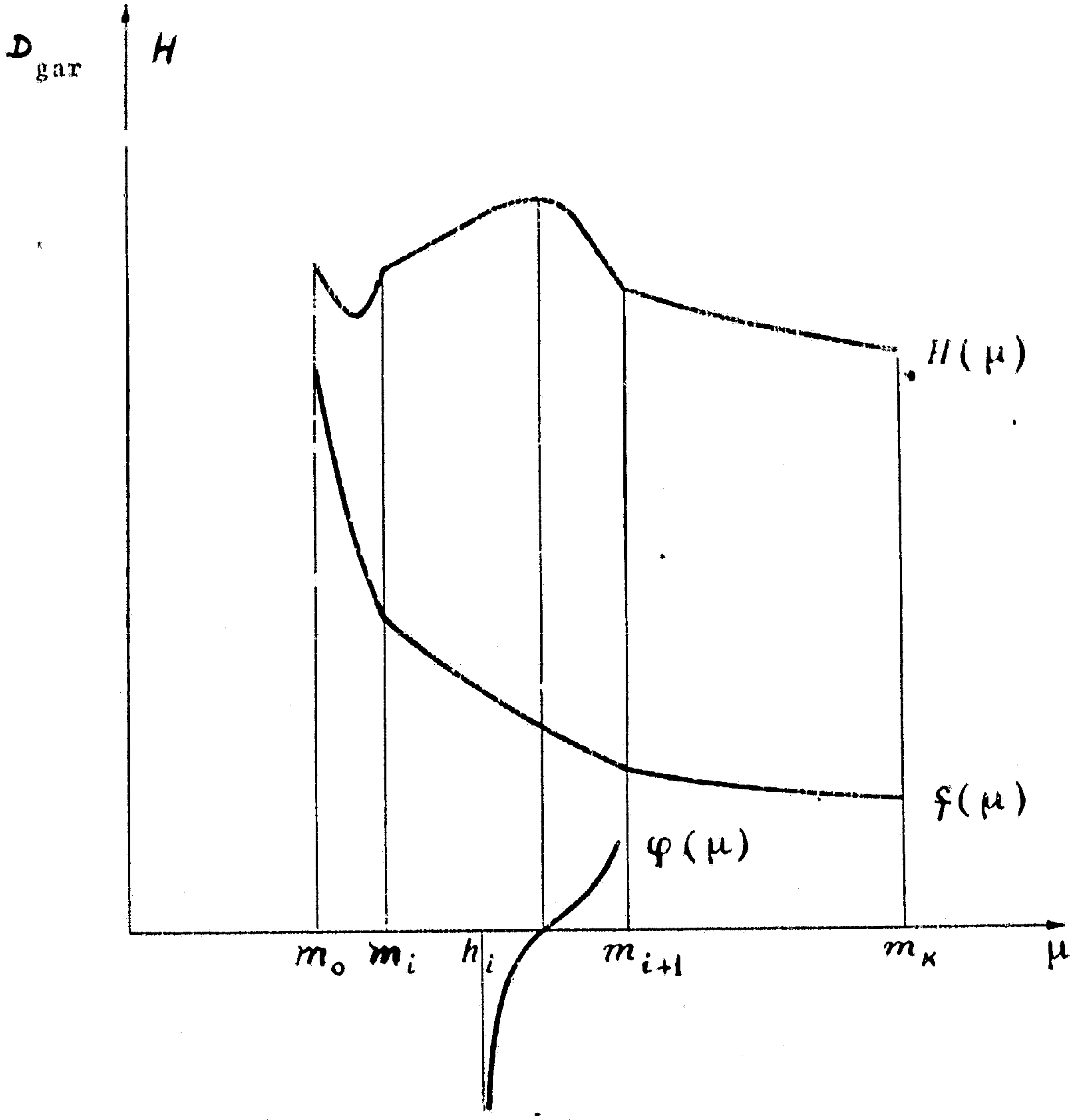

FIG. 2

Illustration of analytical method of finding of max H_{gar}

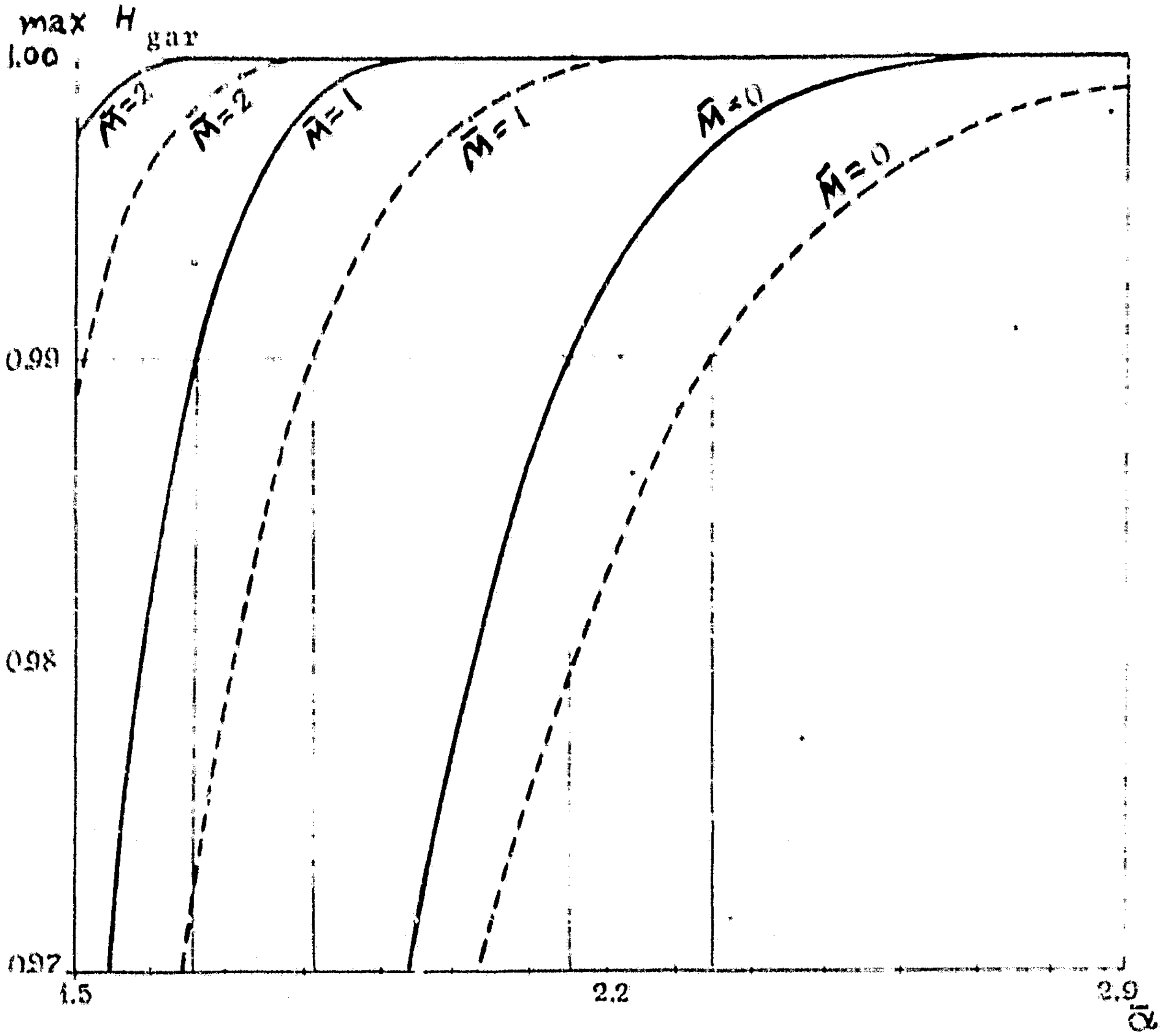

FIG. 3

Dependence of reliability of max H_{gar} on the dimensionless maximum error $\bar{\alpha}$ with various values of the parameter $\bar{M}$.